Algorithms for Determining Tournament Payout Structures

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Algorithms for game and contest management

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100,000s of players, complex tournament structures, real money on the line.

Fantasy Sports: A Quick Review

What are fantasy sports?

Fantasy Sports: A Quick Review

Users “draft” a group of real-world athletes and earn points depending on how well those players perform in games.

BUDGET	AVG. SALARY REMAINING	AVG. FPPG
$52	$17 (3 Players)	14.2

	Pos	Name	FPPG	Salary
	QB	Tom Brady QB HOU @ **NE**, Sat 8:15 PM	21.7	$37
	RB	Le'Veon Bell RB **PIT** @ KC, Sun 1:05 PM	23.9	$41
	RB	Lamar Miller RB **HOU** @ NE, Sat 8:15 PM	12.6	$20
	WR	Select Wide Receiver		

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TEAM MUSCO BOX SCORE				
STARTERS				
SLOT	PLAYER, TEAM POS	OPP	STATUS ET	PTS
QB	Tyrod Taylor, Buf QB Q	@Mia	L 25-28	24.3
QB	Jameis Winston, TB QB	@SF	W 34-17	20.7
RB	C.J. Anderson*, Den RB IR	Hou	W 27-9	16.7
RB	Melvin Gordon, SD RB Q	@Atl	W 33-30	30.1
WR	Brandon Marshall, NYJ WR Q	Bal	W 24-16	3.9
WR	Jarvis Landry, Mia WR	Buf	W 28-25	10.5
TE	Jimmy Graham, Sea TE	@Ari	T 6-6	5.3
FLEX	Mark Ingram, NO RB	@KC	L 21-27	12.2
			TOTAL POINTS:	123.7

IT'S A REBUILDING YEAR BOX SCORE				
STARTERS				
SLOT	PLAYER, TEAM POS	OPP	STATUS ET	PTS
QB	Andrew Luck, Ind QB	@Ten	W 34-26	27.8
QB	Marcus Mariota*, Ten QB IR	Ind	L 26-34	16.7
RB	Frank Gore, Ind RB	@Ten	W 34-26	14.3
RB	Christine Michael, GB RB	Chi	W 26-10	5.5
WR	Jeremy Maclin, KC WR	NO	W 27-21	4
WR	Sammie Coates, Pit WR	NE	L 16-27	0.4
TE	Martellus Bennett, NE TE	@Pit	W 27-16	0.5
FLEX	Spencer Ware, KC RB	NO	W 27-21	19.1
			TOTAL POINTS:	88.3

The Business of Fantasy Sports

Sports covered: American football, baseball, soccer, basketball, hockey, golf, auto racing, mixed martial arts ...

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- 57.4 million users in the US and Canada alone
  + huge international growth
- large platforms run by ESPN, NFL, Yahoo, CBS etc.
- $>$ 60% of participants report watching more games and reading more about sports

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And now you can legally **gamble** on fantasy sports in the US.

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Running contests with 10,000s - 100,000s of players.

Daily Fantasy Sports: Computational Challenges

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- How to evaluate and price athletes? (Anagnostopoulos, Cavallo, Leonardi, Sviridenko, WINE 2016)

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How to distribute prize money amongst top contestants?

Tournament payout structures

1st	$250,000.00
2nd	$100,000.00
3rd	$50,000.00
4th	$25,000.00
5th	$15,000.00
6th	$10,000.00
7th - 8th	$5,000.00
9th - 10th	$4,000.00
11th - 15th	$3,000.00
16th - 20th	$2,000.00
21st - 30th	$1,500.00
31st - 50th	$1,000.00

101st - 150th	$500.00
151st - 200th	$400.00
201st - 300th	$300.00
301st - 400th	$250.00
401st - 500th	$200.00
501st - 800th	$150.00
801st - 1500th	$100.00
1501st - 2500th	$75.00
2501st - 4000th	$60.00
4001st - 6250th	$50.00
6251st - 10000th	$45.00
10001st - 16425th	$40.00

100,000 players $\rightarrow$ $1,000,000 in prizes $\rightarrow$ 10,000 prize winners

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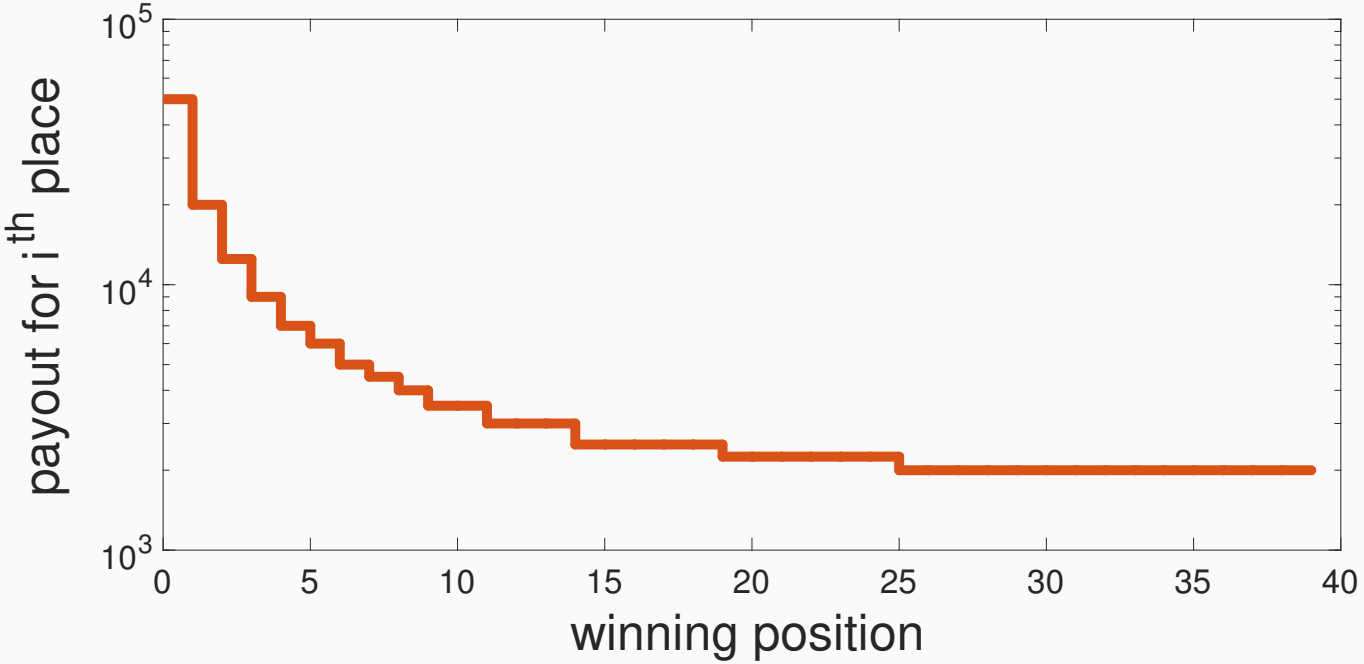
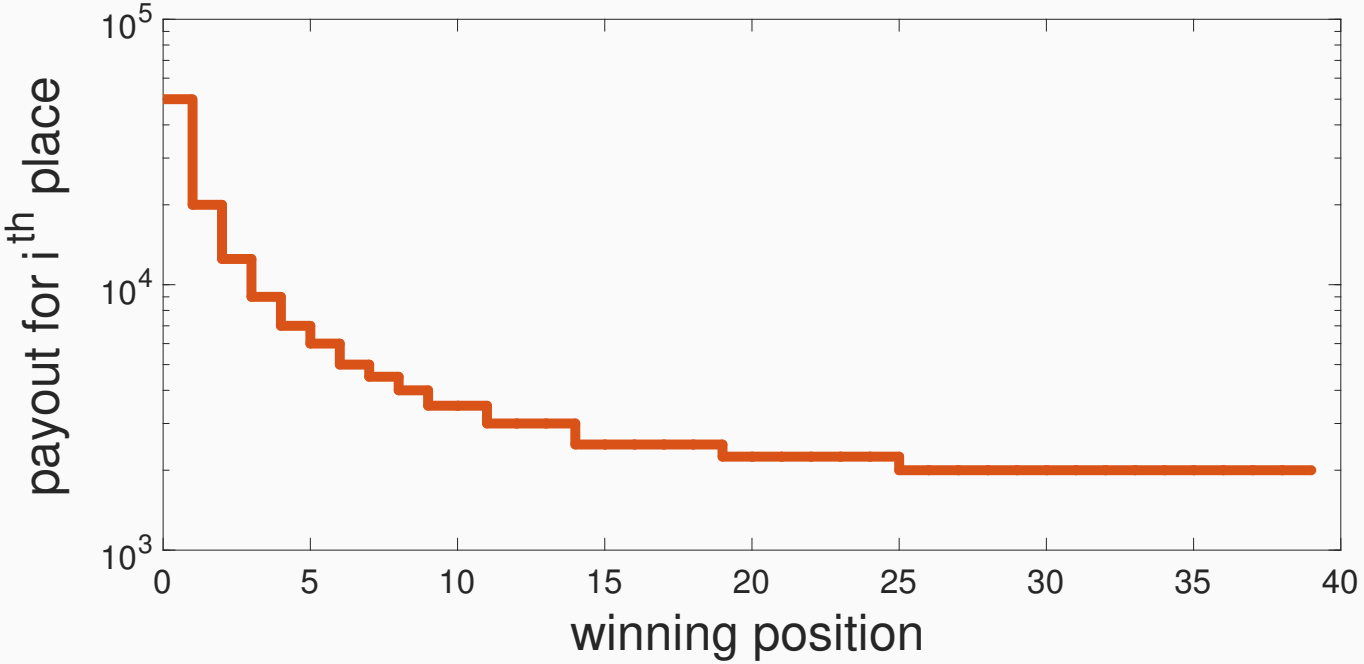


Tournament payout structures

Payouts should:

1. Strongly incentivize players to enter contests.
2. Obey basic aesthetic properties.



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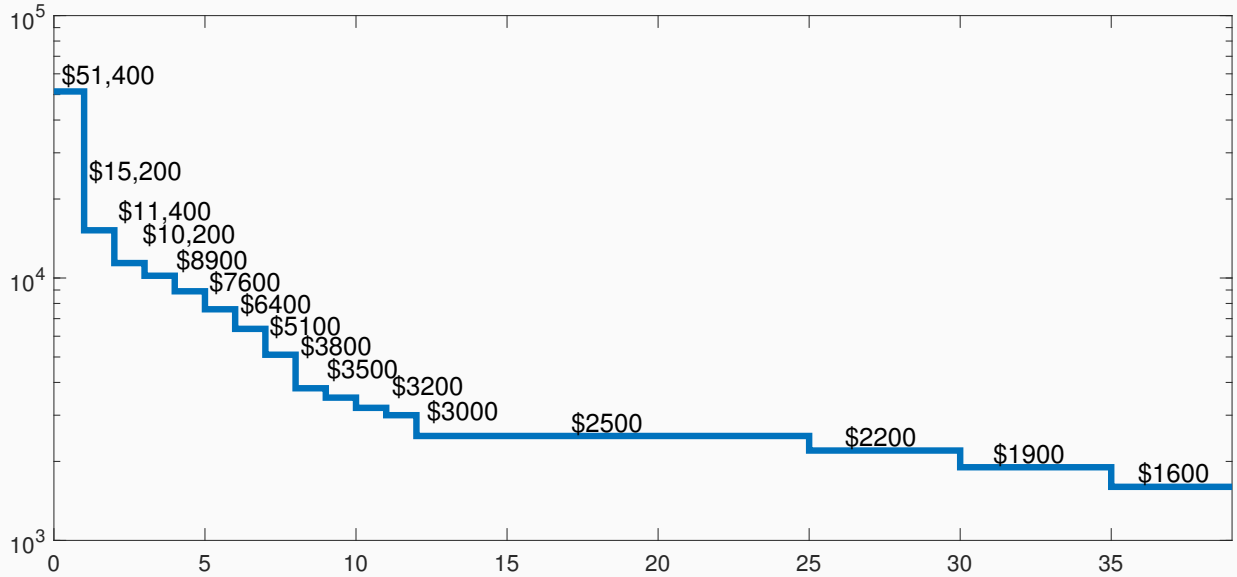

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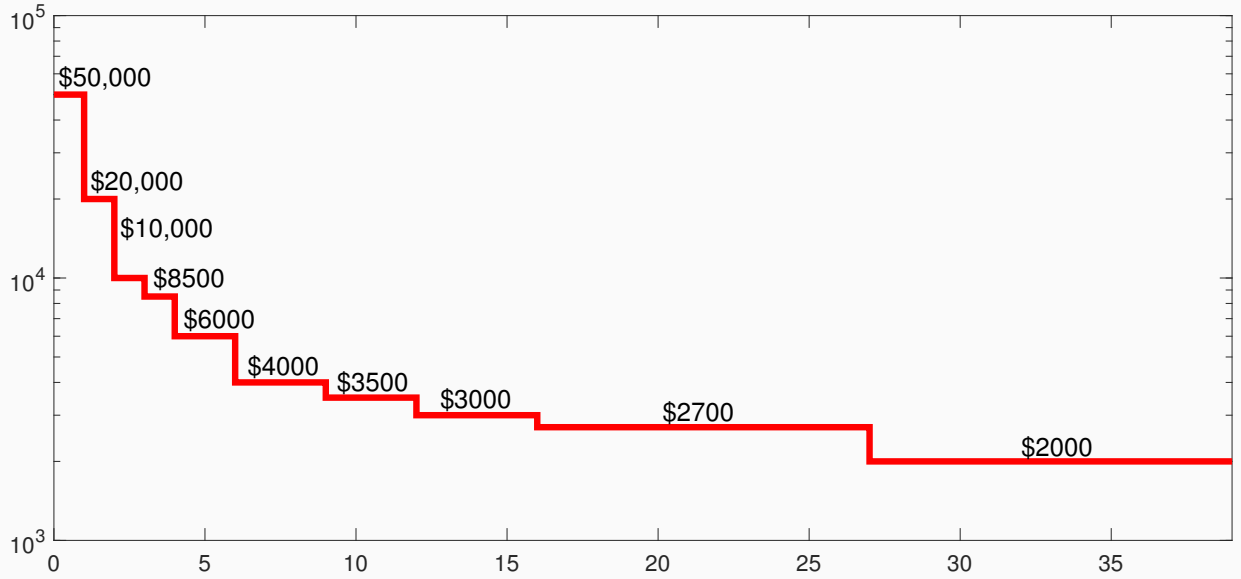

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Prizes need to sum to the total allocated prize pool.

In Daily Fantasy Sports and other large tournaments this is often a strict requirement.

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Daily Fantasy sites run 100s of contests a week, with widely varying entry numbers and prize pools.

Two Step Approach

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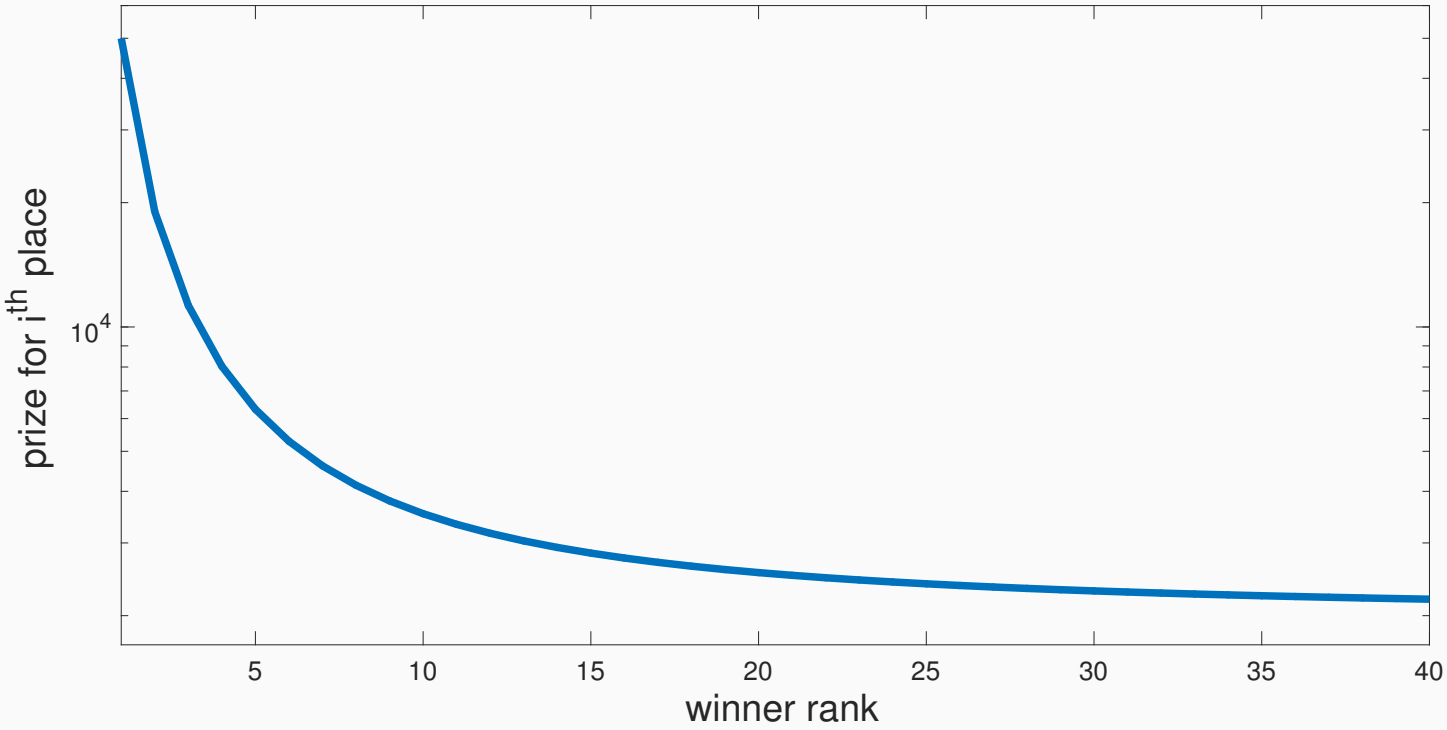
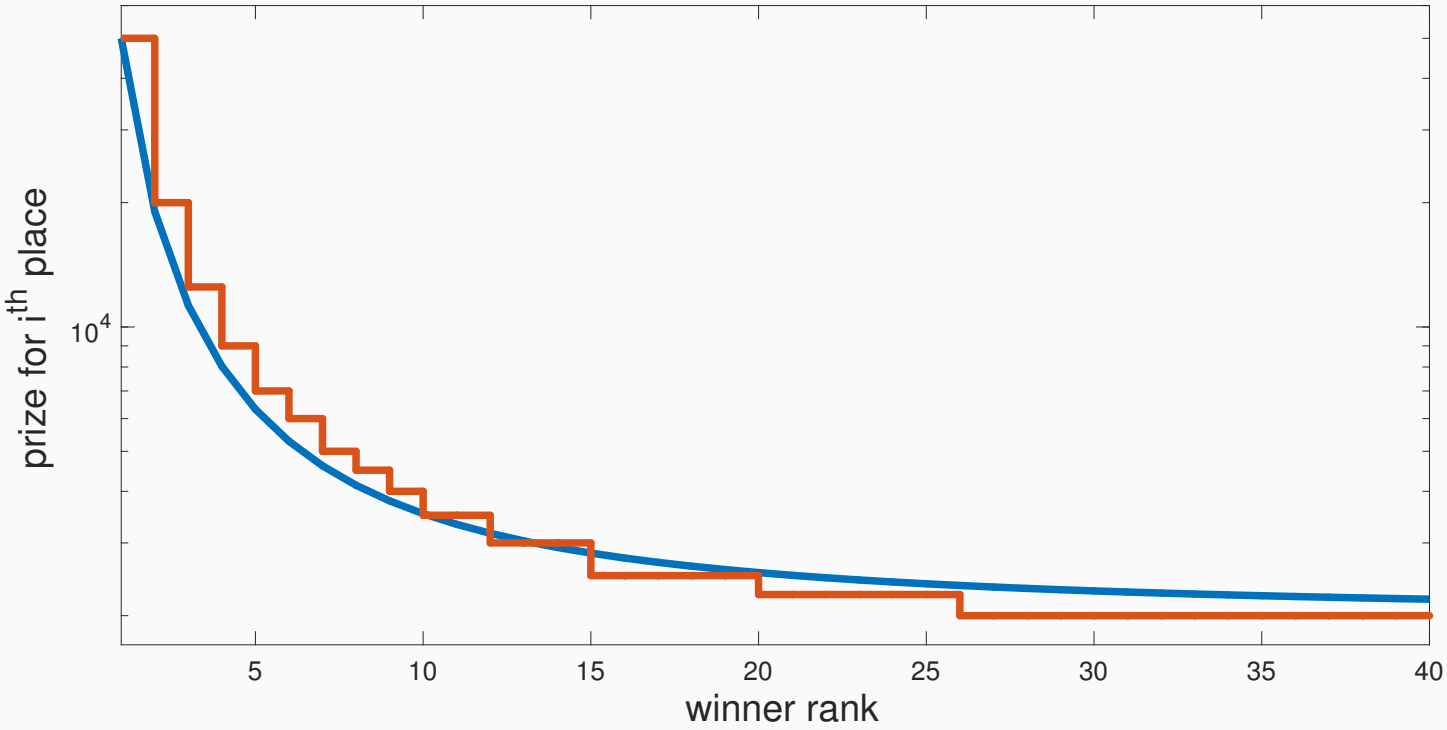


Ideal vs. Rounded payoff structure.

Two Step Approach



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Intermediate prizes defined by simple fall-off function.

Power law Payoffs

We use a power law fall-off:

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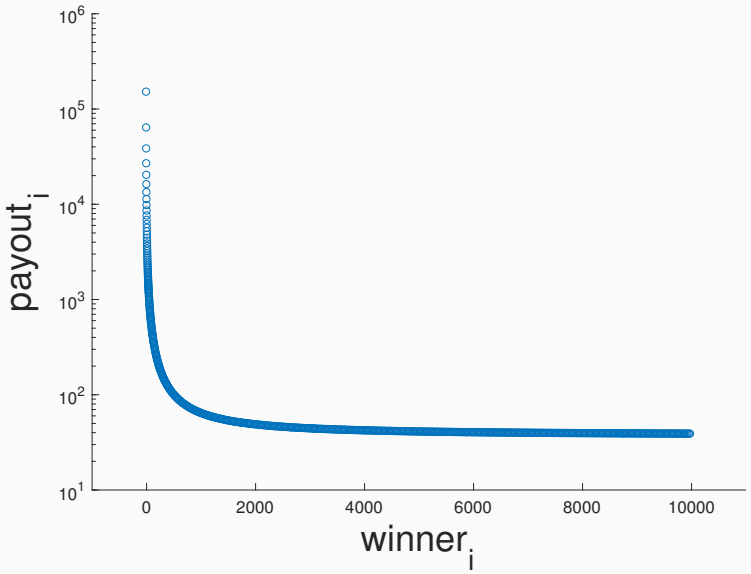

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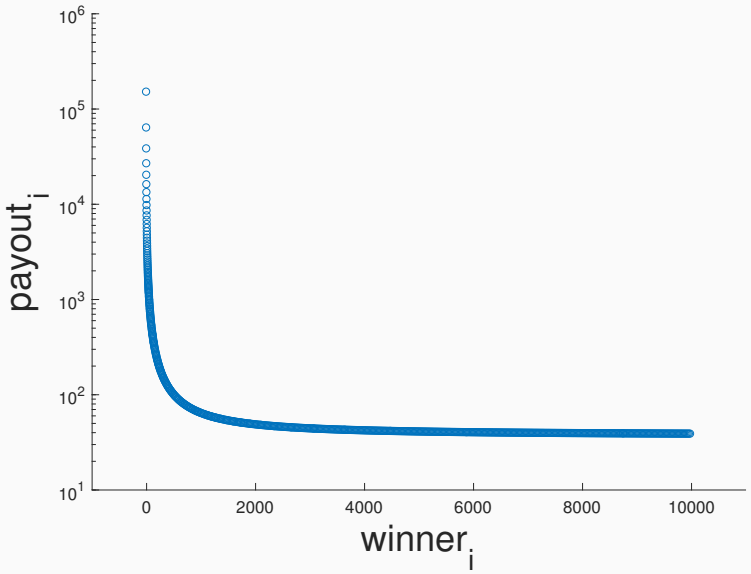

Solve for α such that:

$$\sum_{i=1}^{\text{total winners}} \left(\text{minimum prize} + \frac{\text{top prize - minimum prize}}{i^{\alpha}}\right) = \text{total prize pool.}$$

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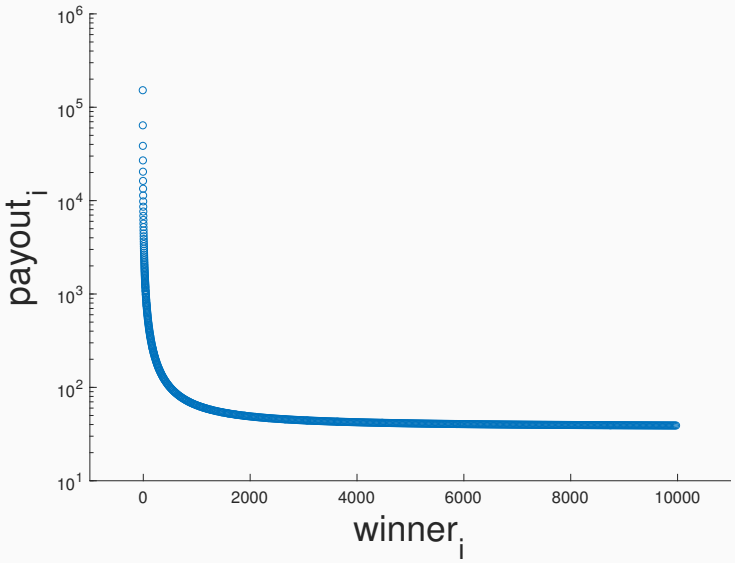
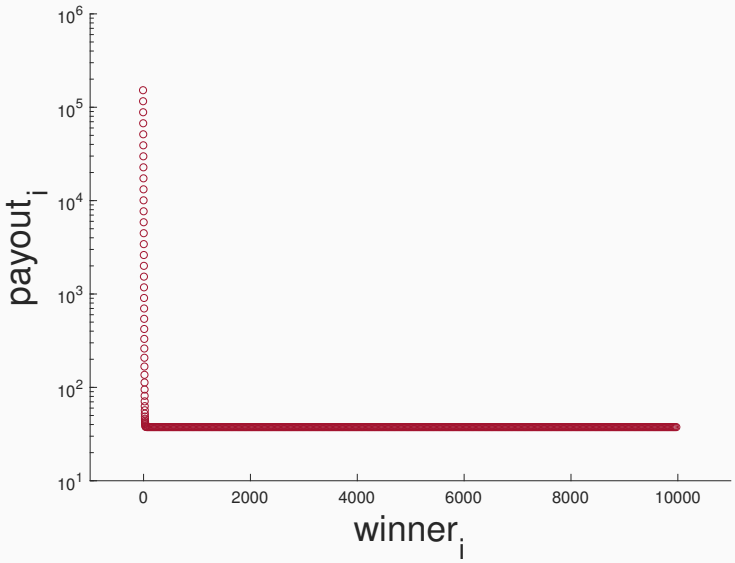


$1/i^{\alpha}$ power law fall-off.



$1/\alpha^{i}$ exponential fall-off.

Why power law?

The Perfect Payout Structure for GPPs

By ganondorf (ganondorf), Last Updated 8 months ago

I feel the need to give a precursor to this post. It may feel like I am critical of some sites in DFS. While this is true, I do not want the impression to be that I am unhappy with them.

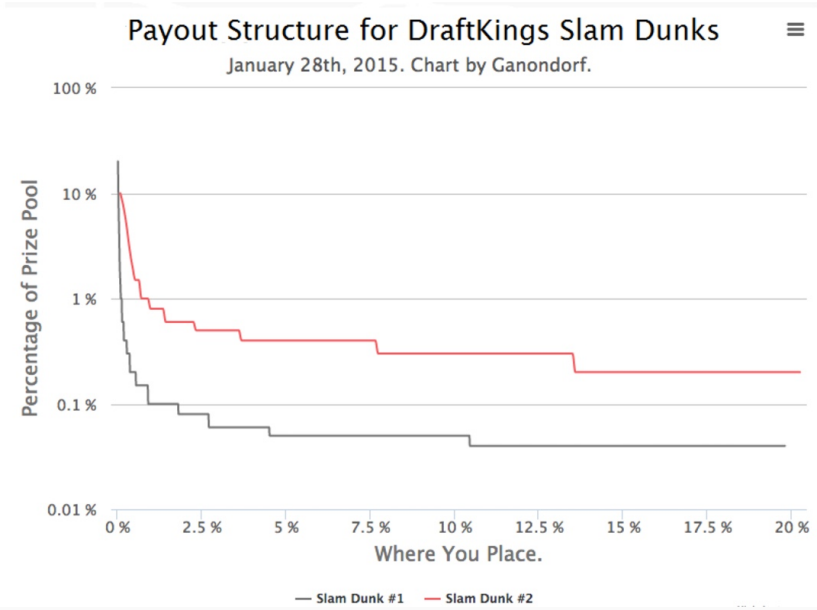
Quite the contrary: I have been very impressed with the growth and advances in the DFS space in the last year. The big sites get A pluses from me. That said, I have some suggestions!

Al Smizzle recently had an insightful tweet:

(Here's a link to the prize structure layout)

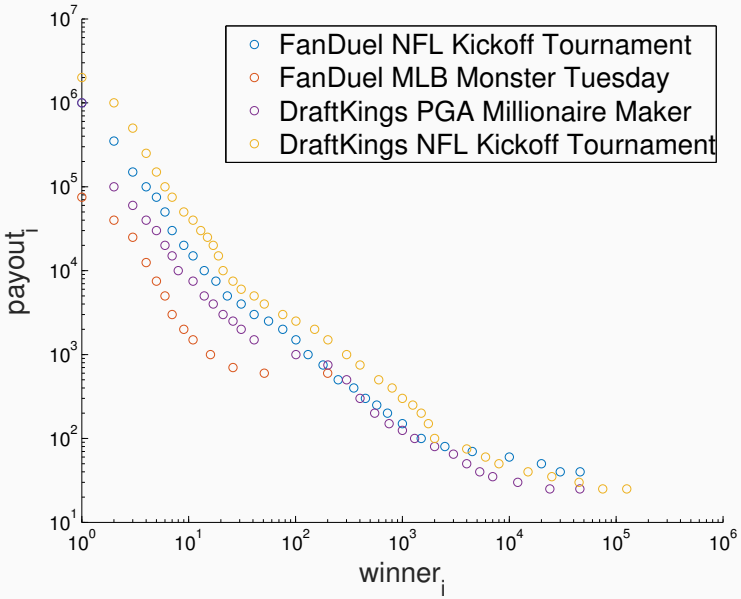
Al, who also discussed prize payout structures on the forums, was referencing DraftKings' Slam Dunk #2. It was a $100,000 prize pool tournament with a $100 buy-in. DraftKings released it on January 28th after their Slam Dunk #1, which had a $500,000 prize pool with a $100 buy-in, filled early. The two contests had a big contrast, which I'd like to demonstrate with a simple chart.

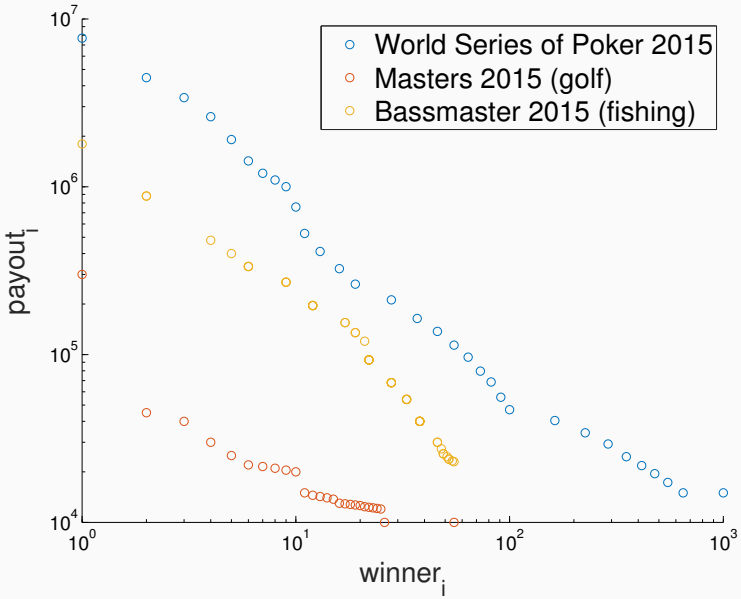
Because we're dealing with percentages, I've changed the scale to be logarithmic. This scale shows the difference between each order of magnitude, e.g. 1% vs. 10%.



Why power law?

Payout distributions for Daily Fantasy Sports and other large tournaments consistent with a power law fall-off.





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Rounding Payouts

Optimization Problem

Input:

Ideal payouts, $\{\pi_1, \ldots, \pi_n\}$.

Output:

Non-overlapping ranges of ranks, $\{S_1, \ldots, S_k\}$.

Prizes $\{P_1, \ldots, P_k\}$

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e.g

Input:

$\{4610, 4138, 3792, 3531, 3327, 3165, 3034, 2925, 2834\}$.

Output:

$\{\{1\}, \{2\}, \{3\}, \{4-5\}\{6-9\}\}$

$\{5000, 4500, 4000, 3500, 3000\}$

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Objective:
minimize $\sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2$

Rounding Constraints

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Such that:

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Option 3: Engineered Heuristic

Matches quality of exactly optimal solutions, scales to very large contests.

Heuristic Algorithm

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3. Merge any buckets with shared prizes and use local swaps to keep bucket sizes monotonic.
4. Spend left-over budget on “singleton buckets”, by violating nice number constraint in a bucket, and as a last resort adding extra winners.

Runtime Results

Integer Program: Only scales to contests with < 100 winners.

Heuristic Algorithm: < 2 second runtimes on a laptop for contests with millions of dollars in prizes, > 10,000 winners.

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(Deployed in production at Yahoo.)

Quantitative Performance

Source	Prize Pool	Top Prize	Min. Prize	# of Winners	# of Buckets	IP Cost	IP Time (ms)	Heur. Cost	Heur. Time (ms)	Heur. Extra Winners
Yahoo	90	25	2	30	7	.89	7.6k	2.35	1	0
Yahoo	180	55	3	30	10	2.82	725k	3.44	1	0
DraftKings	500	100	8	20	10	6.15	2.1k	9.21	1	0
Yahoo	2250	650	150	7	7	32.4	4.0k	187.4	1	0
Yahoo	3000	300	2	850	25	–	–	86.9	7	2
FanDuel	4000	900	50	40	12	20.7	3716k	58.2	2	1
FanDuel	4000	800	75	16	7	46.6	2.9k	230.1	1	4
DraftKings	5000	1250	150	11	8	52.5	6.8k	123.5	1	0
Yahoo	10000	1000	7	550	25	–	–	97.3	8	1
DraftKings	10000	1500	75	42	12	61.3	1291k	173.7	2	0
FanDuel	18000	4000	150	38	10	161.8	131k	347.0	5	0
FanDuel	100000	10000	2	23000	25	–	–	3.1k	152	34
Bassmaster	190700	50000	2000	40	15	–	–	3.5k*	3	0
Bassmaster	190000†	50000	2000	40	15	2.5k	3462k	2.8k	1	0
FLW Fishing	751588	100000	9000	60	25	–	–	6.0k*	3	0
FLW Fishing	751500†	100000	9000	60	25		–	[illegible]	2	0
FanDuel	1000000	100000	15	16000	25	–	–	5.3k	203	7
DraftKings	1000000	100000	5	85000	40	–	–	25.9k	1.2k	0
Bassmaster	1031500	30000	10000	55	25	–	–	13.5k*	14	0
FanDuel	5000000	1000000	40	46000	30	–	–	44.3k	1.0k	0
PGA Golf	9715981	1800000	20000	69	69	–	–	254.5k*	24	0
PGA Golf	1000000†	1800000	20000	75	75	–	–	215.9k*	23	9
DraftKings	10000000	2000000	25	125000	40	–	–	78.7k	1.7k	0
Poker Stars	10393400	1750000	15000	160	25	–	–	133.0k*	27	0
WSOP	60348000	8000000	15000	1000	30	–	–	462.3k*	17	0

Quantitative Performance

Source	Prize Pool	Top Prize	Min. Prize	# of Winners	# of Buckets	IP Cost	IP Time (ms)	Heur. Cost	Heur. Time (ms)	Heur. Extra Winners
Yahoo	90	25	2	30	7	.89	7.6k	2.35	1	0
Yahoo	180	55	3	30	10	2.82	725k	3.44	1	0
DraftKings	500	100	8	20	10	6.15	2.1k	9.21	1	0
Yahoo	2250	650	150	7	7	32.4	4.0k	187.4	1	0
Yahoo	3000	300	2	850	25	–	–	86.9	7	2
FanDuel	4000	900	50	40	12	20.7	3716k	58.2	2	1
FanDuel	4000	800	75	16	7	46.6	2.9k	230.1	1	4
DraftKings	5000	1250	150	11	8	52.5	6.8k	123.5	1	0
Yahoo	10000	1000	7	550	25	–	–	97.3	8	1
DraftKings	10000	1500	75	42	12	61.3	1291k	173.7	2	0
FanDuel	18000	4000	150	38	10	161.8	131k	347.0	5	0
FanDuel	100000	10000	2	23000	25	–	–	3.1k	152	34
Bassmaster	190700	50000	2000	40	15	–	–	3.5k*	3	0
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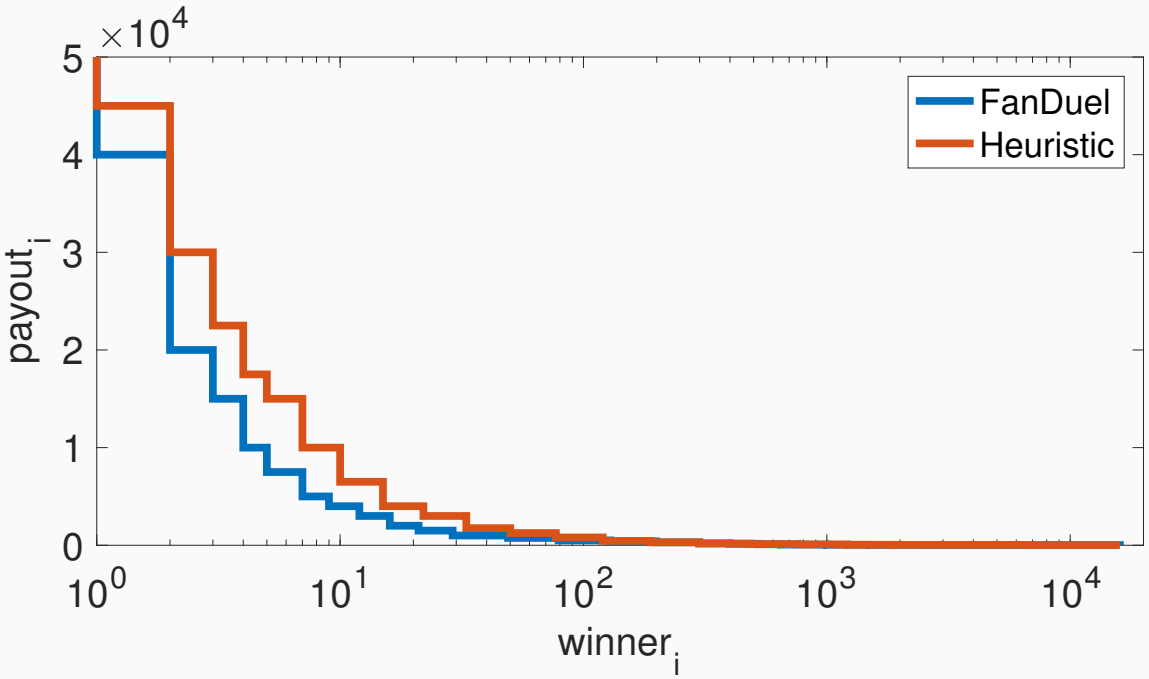
FLW Fishing	751588	100000	9000	60	25	–	–	6.0k*	3	0
FLW Fishing	751500†	100000	9000	60	25		–	[illegible]	2	0
FanDuel	1000000	100000	15	16000	25	–	–	5.3k	203	7
DraftKings	1000000	100000	5	85000	40	–	–	25.9k	1.2k	0
Bassmaster	1031500	30000	10000	55	25	–	–	13.5k*	14	0
FanDuel	5000000	1000000	40	46000	30	–	–	44.3k	1.0k	0
PGA Golf	9715981	1800000	20000	69	69	–	–	254.5k*	24	0
PGA Golf	1000000†	1800000	20000	75	75	–	–	215.9k*	23	9
DraftKings	10000000	2000000	25	125000	40	–	–	78.7k	1.7k	0
Poker Stars	10393400	1750000	15000	160	25	–	–	133.0k*	27	0
WSOP	60348000	8000000	15000	1000	30	–	–	462.3k*	17	0

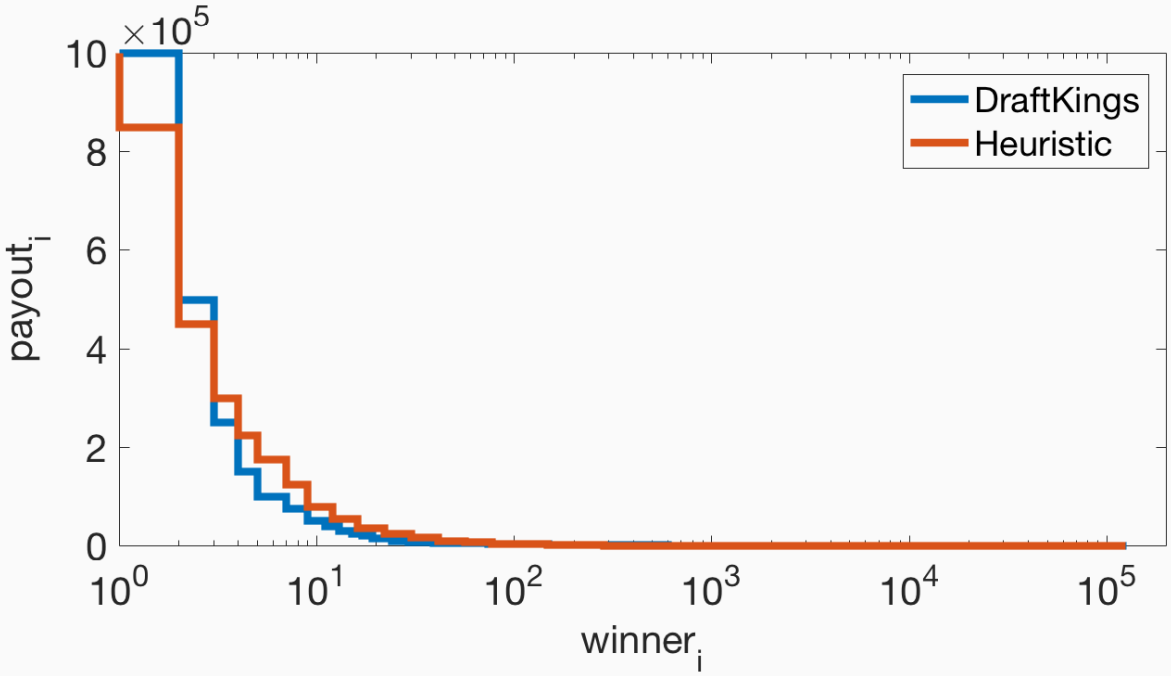
ℓ_2 distance to ideal payouts within 2x-5x that of IP.

Qualitative Performance



FanDuel fantasy football contest

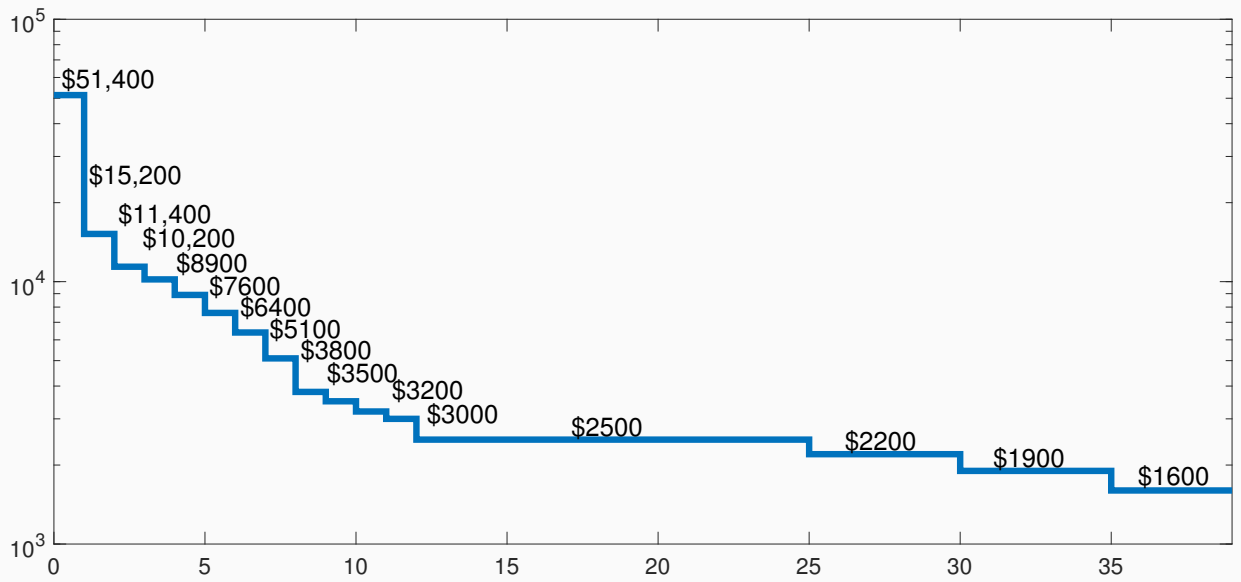
Qualitative Performance



DraftKings fantasy football contest

Qualitative Performance

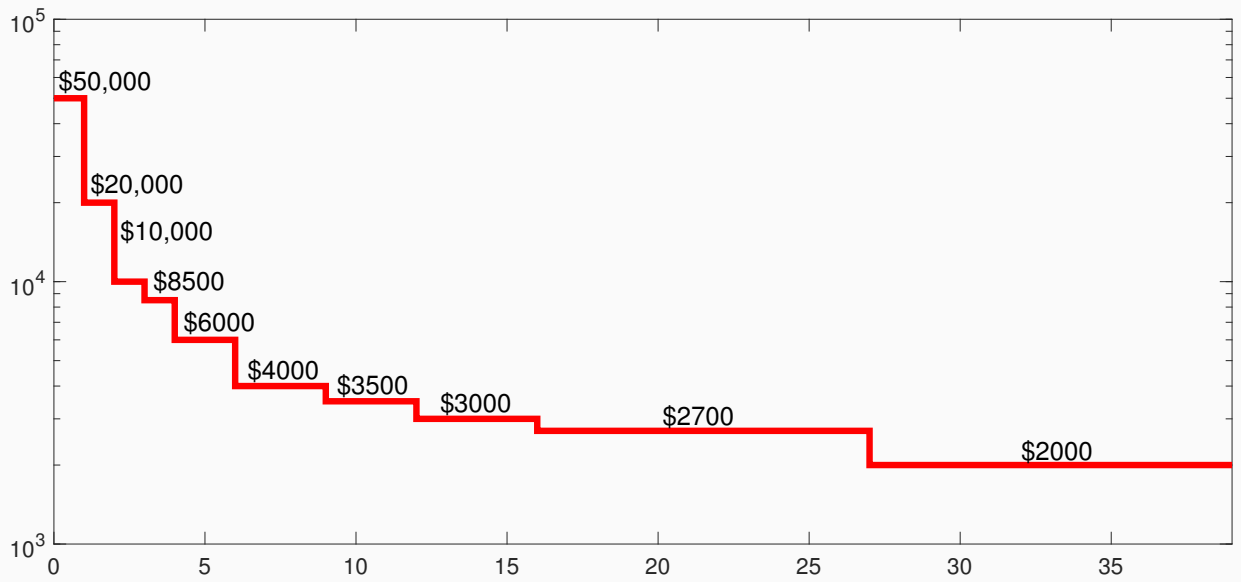
Easily patches “bad” payout structures!



(Bassmaster fishing tournament)

Qualitative Performance

Easily patches “bad” payout structures!



(Bassmaster fishing tournament)

World Series of Poker Payouts

2015 WSOP Payouts		Our Alternative Payouts	
Place	Prize	Place	Prize
1	$7,680,021	1	$8,000,000
2	$4,469,171	2	$4,000,000
3	$3,397,103	3	$2,250,000
4	$2,614,558	4	$1,750,000
5	$1,910,971	5	$1,250,000
6	$1,426,072	6	$1,000,000
7	$1,203,193	7	$950,000
8	$1,097,009	8	$850,000
9	$1,001,020	9	$700,000
10	$756,897	10 - 13	$650,000
11 - 12	$526,778		
13 - 15	$411,453	14 - 17	$500,000
16 - 18	$325,034		
19 - 27	$262,574	18 - 23	$300,000
		24 - 29	$225,000
28 - 36	$211,821	30 - 35	$200,000
36 - 45	$164,086	36 - 42	$150,000
46 - 54	$137,300	43 - 59	$125,000
55 - 63	$113,764		$95,000
64 - 72	$96,445	60 - 77	
73 - 81	$79,668		
82 - 90	$68,624	78 - 99	$75,000
91 - 99	$55,649		
100 - 162	$46,890	100 - 128	$60,000
		128 - 164	$55,000
163 - 225	$40,433	165 - 254	$45,000
226 - 288	$34,157	255 - 345	$35,000
289 - 351	$29,329		
352 - 414	$24,622	346 - 441	$25,000
415 - 477	$21,786		
478 - 549	$19,500	442 - 710	$22,500
550 - 648	$17,282		
649 - 1000	$15,000	711 - 1000	$20,150

Conclusion

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- Lots of interesting algorithmic problems involved in managing massive online tournaments.

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- Lots of interesting algorithmic problems involved in managing massive online tournaments.
- Theoretical formulation leads to provably algorithms as well as practical heuristics.

Thanks!