Algorithms for Determining Tournament Payout Structures

Christopher Musco, Maxim Sviridenko, and Justin Thaler
January 18, 2017

Massachusetts Institute of Technology, Yahoo Research, Georgetown University
There are a lot of interesting algorithmic challenges involved in managing massive online games and contests.
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- online poker tournaments
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100,000s of players, complex tournament structures, real money on the line.
What are fantasy sports?
Fantasy Sports: A Quick Review

Users “draft” a group of real-world athletes and earn points depending on how well those players perform in games.

<table>
<thead>
<tr>
<th>Pos</th>
<th>Name</th>
<th>FPPG</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>QB</td>
<td>Tom Brady</td>
<td>21.7</td>
<td>$37</td>
</tr>
<tr>
<td></td>
<td>QB Hou @ NE, Sat 8:15 PM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>Le’Veon Bell</td>
<td>23.9</td>
<td>$41</td>
</tr>
<tr>
<td></td>
<td>RB Pit @ KC, Sun 1:05 PM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>Lamar Miller</td>
<td>12.6</td>
<td>$20</td>
</tr>
<tr>
<td></td>
<td>RB Hou @ NE, Sat 8:15 PM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Budget: $52
Average Salary Remaining: $17 (3 Players)
Average FPPG: 14.2
Fantasy Sports: A Quick Review

Users “draft” a group of real-world athletes and earn points depending on how well those players perform in games.

### Team Musco Box Score

<table>
<thead>
<tr>
<th>SLOT</th>
<th>PLAYER, TEAM POS</th>
<th>OPP</th>
<th>STATUS ET</th>
<th>PTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>QB</td>
<td>Tyrod Taylor, Buf QB</td>
<td>@Mia</td>
<td>L 25-26</td>
<td>24.3</td>
</tr>
<tr>
<td>QB</td>
<td>Jameis Winston, TB QB</td>
<td>@SF</td>
<td>W 34-17</td>
<td>20.7</td>
</tr>
<tr>
<td>RB</td>
<td>C.J. Anderson*, Den RB</td>
<td>Hou</td>
<td>W 27-9</td>
<td>16.7</td>
</tr>
<tr>
<td>RB</td>
<td>Melvin Gordon, SD RB</td>
<td>@Ati</td>
<td>W 33-30</td>
<td>30.1</td>
</tr>
<tr>
<td>WR</td>
<td>Brandon Marshall, NYJ WR</td>
<td>Bal</td>
<td>W 24-16</td>
<td>3.9</td>
</tr>
<tr>
<td>WR</td>
<td>Jarvis Landry, Mia WR</td>
<td>Buf</td>
<td>W 28-25</td>
<td>10.5</td>
</tr>
<tr>
<td>TE</td>
<td>Jimmy Graham, Sea TE</td>
<td>@Ari</td>
<td>T 6-6</td>
<td>5.3</td>
</tr>
<tr>
<td>FLEX</td>
<td>Mark Ingram, NO RB</td>
<td>@KC</td>
<td>L 21-27</td>
<td>12.2</td>
</tr>
</tbody>
</table>

**TOTAL POINTS:** 123.7

### It’s a Rebuilding Year Box Score

<table>
<thead>
<tr>
<th>SLOT</th>
<th>PLAYER, TEAM POS</th>
<th>OPP</th>
<th>STATUS ET</th>
<th>PTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>QB</td>
<td>Andrew Luck, Ind QB</td>
<td>@Ten</td>
<td>W 34-26</td>
<td>27.8</td>
</tr>
<tr>
<td>QB</td>
<td>Marcus Mariota*, Ten QB</td>
<td>Ind</td>
<td>L 26-34</td>
<td>16.7</td>
</tr>
<tr>
<td>RB</td>
<td>Frank Gore, Ind RB</td>
<td>@Ten</td>
<td>W 34-26</td>
<td>14.3</td>
</tr>
<tr>
<td>RB</td>
<td>Christine Michael, GB RB</td>
<td>Chi</td>
<td>W 26-10</td>
<td>5.5</td>
</tr>
<tr>
<td>WR</td>
<td>Jeremy Maclin, KC WR</td>
<td>NO</td>
<td>W 27-21</td>
<td>4</td>
</tr>
<tr>
<td>WR</td>
<td>Sammie Coates, Pit WR</td>
<td>NE</td>
<td>L 16-27</td>
<td>0.4</td>
</tr>
<tr>
<td>TE</td>
<td>Martellus Bennett, NE TE</td>
<td>@Pit</td>
<td>W 27-16</td>
<td>0.5</td>
</tr>
<tr>
<td>FLEX</td>
<td>Spencer Ware, KC RB</td>
<td>NO</td>
<td>W 27-21</td>
<td>19.1</td>
</tr>
</tbody>
</table>

**TOTAL POINTS:** 88.3
Sports covered: American football, baseball, soccer, basketball, hockey, golf, auto racing, mixed martial arts ...
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- 57.4 million users in the US and Canada alone
  + huge international growth
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- large platforms run by ESPN, NFL, Yahoo, CBS etc.
The Business of Fantasy Sports

Sports covered: American football, baseball, soccer, basketball, hockey, golf, auto racing, mixed martial arts ...

- 57.4 million users in the US and Canada alone
  + huge international growth
- large platforms run by ESPN, NFL, Yahoo, CBS etc.
- > 60% of participants report watching more games and reading more about sports
And now you can legally \textit{gamble} on fantasy sports in the US.
And now you can legally **gamble** on fantasy sports in the US.

Led to emergence of **Daily Fantasy Sports**.
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Running contests with 10,000s - 100,000s of players.
Daily Fantasy Sports: Computational Challenges

• How to evaluate and price athletes? (Anagnostopoulos, Cavallo, Leonardi, Sviridenko, WINE 2016)
• How to structure scoring to manage competition variance?
• How to ensure fairness and effectively separate new players from “sharks”? (see NY Times article)
• How to distribute prize money amongst top contestants?
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How to distribute prize money amongst top contestants?
## Tournament Payout Structures

<table>
<thead>
<tr>
<th>Position</th>
<th>Prize Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>$250,000.00</td>
</tr>
<tr>
<td>2nd</td>
<td>$100,000.00</td>
</tr>
<tr>
<td>3rd</td>
<td>$50,000.00</td>
</tr>
<tr>
<td>4th</td>
<td>$25,000.00</td>
</tr>
<tr>
<td>5th</td>
<td>$15,000.00</td>
</tr>
<tr>
<td>6th</td>
<td>$10,000.00</td>
</tr>
<tr>
<td>7th - 8th</td>
<td>$5,000.00</td>
</tr>
<tr>
<td>9th - 10th</td>
<td>$4,000.00</td>
</tr>
<tr>
<td>11th - 15th</td>
<td>$3,000.00</td>
</tr>
<tr>
<td>16th - 20th</td>
<td>$2,000.00</td>
</tr>
<tr>
<td>21st - 30th</td>
<td>$1,500.00</td>
</tr>
<tr>
<td>31st - 50th</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>101st - 150th</td>
<td>$500.00</td>
</tr>
<tr>
<td>151st - 200th</td>
<td>$400.00</td>
</tr>
<tr>
<td>201st - 300th</td>
<td>$300.00</td>
</tr>
<tr>
<td>301st - 400th</td>
<td>$250.00</td>
</tr>
<tr>
<td>401st - 500th</td>
<td>$200.00</td>
</tr>
<tr>
<td>501st - 800th</td>
<td>$150.00</td>
</tr>
<tr>
<td>801st - 1500th</td>
<td>$100.00</td>
</tr>
<tr>
<td>1501st - 2500th</td>
<td>$75.00</td>
</tr>
<tr>
<td>2501st - 4000th</td>
<td>$60.00</td>
</tr>
<tr>
<td>4001st - 6250th</td>
<td>$50.00</td>
</tr>
<tr>
<td>6251st - 10000th</td>
<td>$45.00</td>
</tr>
<tr>
<td>10001st - 16425th</td>
<td>$40.00</td>
</tr>
</tbody>
</table>

100,000 players → $1,000,000 in prizes → 10,000 prize winners
Tournament payout structures

Payouts should:

1. Strongly incentivize players to enter contests.
2. Obey basic aesthetic properties.
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     \{10, 15, \ldots, 95,100,125, 150, \ldots, 225, 250, 300, 350, \ldots, 950, 1000\}
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   • Are “nice numbers” ($1000 is preferable to $1012.15)
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   • Fall into manageable number of buckets (i.e. 25 – 40)
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(Bassmaster fishing tournament)
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Prizes need to sum to the total allocated prize pool.
In Daily Fantasy Sports and other large tournaments this is often a strict requirement.
How hard is it to construct payout structures by hand?
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Very difficult! Even for just a single contest.
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World Series of Poker organizers apparently struggled with the problem for years before commissioning their own algorithm.
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**Very difficult!** Even for just a single contest.

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Daily Fantasy sites run 100s of contests a week, with widely varying entry numbers and prize pools.
Two Step Approach

Payouts should:

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Two step approach:

1. Choose “ideal payouts” that don’t satisfy aesthetics.
2. Round to a payout structure that does.
TWO STEP APPROACH

Ideal vs. Rounded payoff structure.
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Step 1: Ideal Payouts

Fix top prize, minimum prize, and number of prize winners.
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Intermediate prizes defined by simple fall-off function.
We use a **power law** fall-off:

$i^{th}$ prize proportional to $1/i^\alpha$, for constant $\alpha$. 
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\[ i^{th} \text{ prize proportional to } 1/i^\alpha, \text{ for constant } \alpha. \]
We use a power law fall-off:

\( i^{\text{th}} \) prize proportional to \( 1/i^\alpha \), for constant \( \alpha \).

Solve for \( \alpha \) such that:

\[
\sum_{i=1}^{\text{total winners}} \left( \text{minimum prize} + \frac{\text{top prize} - \text{minimum prize}}{i^\alpha} \right) = \text{total prize pool.}
\]
Why power law?

A power law richly rewards the best players, but ensures lower winners still receive substantial prizes.
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$1/i^\alpha$ power law fall-off. $1/\alpha^i$ exponential fall-off.
Why Power Law?

The Perfect Payout Structure for GPPs

By ganondorf (ganondorf), Last Updated 8 months ago

I feel the need to give a precursor to this post. It may feel like I am critical of some sites in DFS. While this is true, I do not want the impression to be that I am unhappy with them. Quite the contrary: I have been very impressed with the growth and advances in the DFS space in the last year. The big sites get A pluses from me. That said, I have some suggestions!

Al Smizzle recently had an insightful tweet:

Al Smizzle (@AlSmizzle)

My favorite GPP payout structure ever. 2nd is 75% of 1st, and 10th is 10% of 1st. Can we make this the standard? evernote.com/shard/s481/shv/

(Here's a link to the prize structure layout)

Al, who also discussed prize payout structures on the forums, was referencing DraftKings' Slam Dunk #2. It was a $100,000 prize pool tournament with a $100 buy-in. DraftKings released it on January 28th after their Slam Dunk #1, which had a $500,000 prize pool with a $100 buy-in, filled early. The two contests had a big contrast, which I'd like to demonstrate with a simple chart.

Because we're dealing with percentages, I've changed the scale to be logarithmic. This scale shows the difference between each order of magnitude, e.g. 1% vs. 10%.

Payout Structure for DraftKings Slam Dunks

Why power law?

Payout distributions for Daily Fantasy Sports and other large tournaments consistent with a power law fall-off.
Two Step Approach

Payouts should:

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Rounding Payouts

Optimization Problem

Input:
Ideal payouts, \( \{\pi_1, \ldots, \pi_n\} \).

Output:
Non-overlapping ranges of ranks, \( \{S_1, \ldots, S_k\} \).
Prizes \( \{P_1, \ldots, P_k\} \)
Optimization Problem

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Prizes \( \{P_1, \ldots, P_k\} \)

e.g

Input:
\( \{4610, 4138, 3792, 3531, 3327, 3165, 3034, 2925, 2834\} \).

Output:
\( \{\{1\}, \{2\}, \{3\}, \{4 - 5\},\{6 - 9\}\} \)
\( \{5000, 4500, 4000, 3500, 3000\} \)
Rounding Payouts

Optimization Problem

Input:
Ideal payouts, \( \{\pi_1, \ldots, \pi_n\} \).

Output:
Non-overlapping ranges of ranks, \( \{S_1, \ldots, S_k\} \).
Prizes \( \{P_1, \ldots, P_k\} \)

Objective:
minimize \( \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2 \)
minimize $\sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2$
Rounding Constraints

\[ \text{minimize } \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2 \]

Such that:

- \( P_i \) is a nice number
minimize \[ \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2 \]

Such that:

- \( P_i \in \{100, 200, 300, 400, 500, 1000, \ldots, 10000, 15000, \ldots\} \)
Rounding Constraints

\[
\text{minimize } \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2
\]

Such that:

- \( P_i \in \{100, 200, 300, 400, 500, 1000, \ldots, 10000, 15000, \ldots \} \)
- \( P_1 > P_2 > \ldots > P_k \geq \text{minimum prize} \)
minimize $\sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2$

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- $P_1 > P_2 > \ldots > P_k \geq$ minimum prize
- $\sum_{i=1}^{k} |S_i| = n$
minimize \( \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2 \)

Such that:

- \( P_i \in \{100, 200, 300, 400, 500, 1000, \ldots, 10000, 15000, \ldots\} \)
- \( P_1 > P_2 > \ldots > P_k \geq \) minimum prize
- \( \sum_{i=1}^{k} |S_i| = n \)
- \( \sum_{i=1}^{k} |S_i| \cdot P_i = B \) (total prize pool)
Rounding Constraints

\[
\text{minimize } \sum_{i=1}^{k} \sum_{j \in S_i} (\pi_j - P_j)^2
\]

Such that:

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- \( P_1 > P_2 > \ldots > P_k \geq \text{minimum prize} \)
- \( \sum_{i=1}^{k} |S_i| = n \)
- \( \sum_{i=1}^{k} |S_i| \cdot P_i = B \) (total prize pool)
- \( |S_1| \leq |S_2| \leq \ldots \leq |S_k| \)
How can we solve this optimization problem?

Option 1: Multi-dimensional dynamic programming $O(kn^2B\log B)$ time if there are $O(\log B)$ “nice numbers” $B$.

Option 2: Integer Program Off-the-shelf solver (GLPK) works well for relatively small contests.

Option 3: Engineered Heuristic Matches quality of exactly optimal solutions, scales to very large contests.
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Matches quality of exactly optimal solutions, scales to very large contests.
Outline of heuristic algorithm:
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1. Set initial bucket sizes to increase according to a power law, distributing $n$ places amongst $k$ buckets.
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3. Merge any buckets with shared prizes and use local swaps to keep bucket sizes monotonic.
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2. Choose initial prize $P_i$ to be the largest nice number smaller than mean of ideal prizes in bucket $S_i$.

3. Merge any buckets with shared prizes and use local swaps to keep bucket sizes monotonic.

4. Spend left-over budget on “singleton buckets”, by violating nice number constraint in a bucket, and as a last resort adding extra winners.
Runtime Results

**Integer Program**: Only scales to contests with < 100 winners.

**Heuristic Algorithm**: < 2 second runtimes on a laptop for contests with millions of dollars in prizes, > 10,000 winners.
Runtime Results

Integer Program: Only scales to contests with < 100 winners.

Heuristic Algorithm: < 2 second runtimes on a laptop for contests with millions of dollars in prizes, > 10,000 winners.

(Deployed in production at Yahoo.)
## Quantitative Performance

<table>
<thead>
<tr>
<th>Source</th>
<th>Prize Pool</th>
<th>Top Prize</th>
<th>Min. Prize</th>
<th># of Winners</th>
<th># of Buckets</th>
<th>IP Cost</th>
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\[ \ell_2 \text{ distance to ideal payouts within 2x-5x that of IP.} \]
Qualitative Performance

FanDuel fantasy football contest
DraftKings fantasy football contest
Easily patches “bad” payout structures!

(Bassmaster fishing tournament)
Easily patches “bad” payout structures!

(Bassmaster fishing tournament)
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• Lots of interesting algorithmic problems involved in managing massive online tournaments.
• Theoretical formulation leads to provably algorithms as well as practical heuristics.

Thanks!
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Thanks!